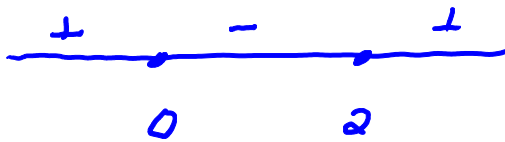


Suppose the function  $f$  is defined by  $f(x) = x^3 - 3x^2 + 1$ .

a) On what intervals is the function  $f$  increasing? Decreasing?

Mean Value Theorem

$$f'(x) = 3x^2 - 6x = 3x(x-2).$$



So  $f$  is increasing on  $(-\infty, 0) \cup (2, \infty)$ .

$f$  is decreasing on  $(0, 2)$ .

b) Determine the local extreme values of  $f$ .


$f$  has a local maximum at  $x = 0$ .



$f$  has a local minimum at  $x = 2$ .



c) Determine the intervals on which  $f$  is concave upwards or concave downwards. Locate any points of inflection.

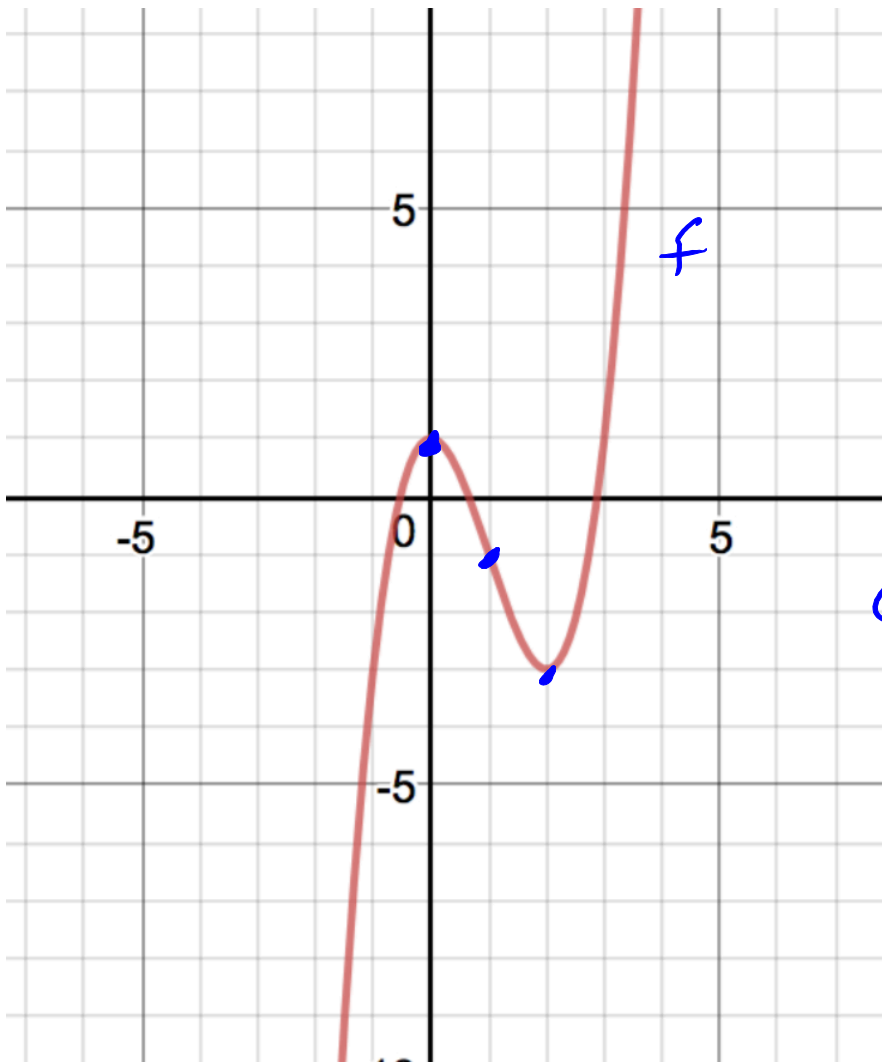
$$f''(x) = 6x - 6 = 6(x-1)$$


The graph of  $f$  is concave up  
on  $(1, \infty)$ . The graph of  $f$   
is concave down  $(-\infty, 1)$

d) Sketch the graph of  $f$ .

Desmos: <https://www.desmos.com/calculator>





$(1, f(1))$

$(1, -1)$

inflection  
pt

.